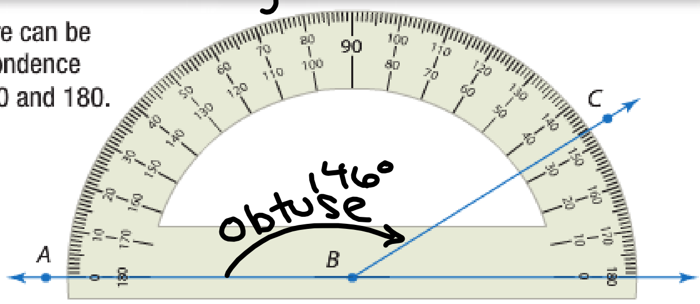
When we want to prove that certain relationships exist among angles, the following two postulates could be useful:

Postulate 2.10 Protractor Postulate → an angle can be measured.

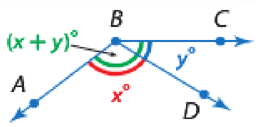
Words Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

Example If $\overrightarrow{BA}$ is placed along the protractor at 0°, then the measure of $\angle ABC$ corresponds to a positive real number.



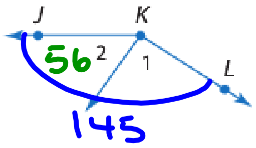
Postulate 2.11 Angle Addition Postulate

D is in the interior of $\angle ABC$ if and only if
$m\angle ABD + m\angle DBC = m\angle ABC$.



Example 1:

Find $m\angle 1$ if $m\angle 2 = 56$ and $m\angle JKL = 145$.

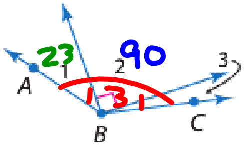


$$m\angle 1 + m\angle 2 = m\angle JKL$$
$$m\angle 1 + 56 = 145$$
$$-56 \mid -56$$
$$\boxed{m\angle 1 = 89}$$

Example 2:

informal proof

If $m\angle 1 = 23$ and $m\angle ABC = 131$, find $m\angle 3$. Justify each step.



$m\angle ABC = m\angle 1 + m\angle 2 + m\angle 3$ Angle Addition Post.

$131 = 23 + 90 + m\angle 3$ substitution

$131 = 113 + m\angle 3$ substitution

$-113 \quad -113$ subtraction property

$18 = m\angle 3$ substitution

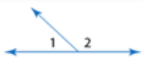
StudyTip

Linear Pair Theorem The Supplement Theorem may also be known as the *Linear Pair Theorem.*

already know

Theorems

2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.



Example $m\angle 1 + m\angle 2 = 180$

2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Example $m\angle 1 + m\angle 2 = 90$

} adjacent angles

Example 3:

$\angle 6$ and $\angle 7$ form a linear pair. If $m\angle 6 = 3x + 32$ and $m\angle 7 = 5x + 12$, find x, $m\angle 6$, and $m\angle 7$.

$m\angle 6 + m\angle 7 = 180$

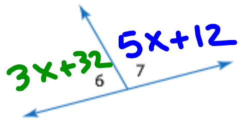
$3x + 32 + 5x + 12 = 180$

$8x + 44 = 180$

$-44 \quad -44$

$\frac{8x}{8} = \frac{136}{8}$

$x = 17$



$m\angle 6 = 3x + 32$

$= 3(17) + 32$

$51 + 32$

$m\angle 6 = 83$

$m\angle 7 = 5x + 12$

$= 5(17) + 12$

$= 85 + 12$

$m\angle 7 = 97$

check:

$m\angle 6 + m\angle 7 = 180$

$83 + 97 = 180$

$180 = 180$ ✓

Theorem 2.5 Properties of Angle Congruence

Reflexive Property of Congruence

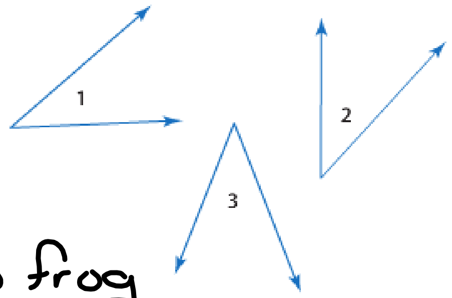
$\angle 1 \cong \angle 1$ an angle is $\cong$ to itself.

Symmetric Property of Congruence

If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$. swap

Transitive Property of Congruence

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$. leap frog



Algebraic properties can be applied to prove theorems for congruence relationships involving complementary and supplementary angles:

ReadingMath

Abbreviations and Symbols

The notation ∡ means *angles*.

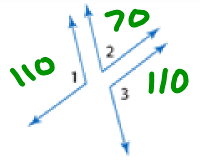
Theorems

2.6 Congruent Supplements Theorem

Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation *∡ suppl. to same ∠ or ≅ ∡ are ≅.*

Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.

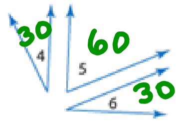


2.7 Congruent Complements Theorem

Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation *∡ compl. to same ∠ or ≅ ∡ are ≅.*

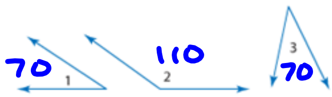
Example If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.



Example 4:

Given: $\angle 1$ and $\angle 2$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

Prove: $\angle 1 \cong \angle 3$

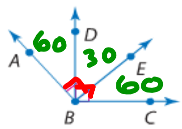


STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are supplementary $\angle 2$ and $\angle 3$ are supplementary	1. Given
2. $m\angle 1 + m\angle 2 = 180$ $m\angle 2 + m\angle 3 = 180$	2. Def. of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ $-m\angle 2 \quad -m\angle 2$	3. Substitution
4. $m\angle 2 = m\angle 2$	4. Reflexive Prop. of =
5. $m\angle 1 = m\angle 3$	5. Subtraction Prop.
6. $\angle 1 \cong \angle 3$	6. Def. of congruence

Example 5:

In the figure, $\angle ABE$ and $\angle DBC$ are right angles. Prove that $\angle ABD \cong \angle EBC$.

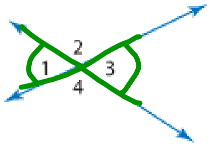
Given



STATEMENTS	REASONS
1. $\angle ABE$ and $\angle DBC$ are right angles	1. Given
2. $m\angle ABE = 90$ $m\angle DBC = 90$	2. Def. of right angle
3. $m\angle ABD$ and $m\angle DBE$ are complementary, $m\angle DBE$ and $m\angle EBC$ are complementary	3. Complement Thm.
4. $\angle ABD \cong \angle EBC$	4. Congruent Complements Thm.

Theorem 2.8 **Vertical Angles Theorem** BOW-TIES

If two angles are vertical angles, then they are congruent.



Abbreviation *Vert. ∡ are ≅.*

Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

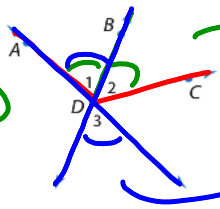
Vertical angles are formed by intersecting lines (think of an X).

Example 6:

Prove that if $\overrightarrow{DB}$ bisects $\angle ADC$, then $\angle 2 \cong \angle 3$.

Given: $\overrightarrow{DB}$ bisects $\angle ADC$. (angle bisector)

Prove: $\angle 2 \cong \angle 3$



$\angle 1 \cong \angle 2$

$\angle 1 \cong \angle 3$

STATEMENTS	REASONS
1. $\overline{DB}$ bisects $\angle ADC$.	1. Given
2. $\angle 1 \cong \angle 2$	2. Def. of Angle Bisector
3. $\angle 1$ and $\angle 3$ are vertical angles	3. Def. of Vertical Angles
4. $\angle 3 \cong \angle 1$	4. Vertical Angles Thm.
5. $\angle 3 \cong \angle 2$ ($\angle 3 \cong \boxed{\angle 1, \angle 1} \cong \angle 2$)	5. Transitive Prop (or Substitution)
6. $\angle 2 \cong \angle 3$	6. Symmetric Prop.

Example 7:

Given Given

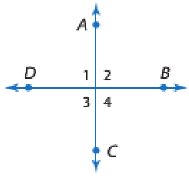
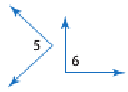
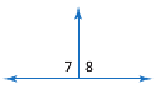
If $\angle 3$ and $\angle 4$ are vertical angles, $m\angle 3 = 6x + 2$, and $m\angle 4 = 8x - 14$, find $m\angle 3$ and $m\angle 4$. Justify each step.

1 $\angle 3 \cong \angle 4$	1 Vertical Angles Thm.
2 $m\angle 3 = m\angle 4$	2 Def. of congruence
3 $6x + 2 = 8x - 14$	3 Substitution
4 $-6x \quad -6x$	4 Subtraction Prop.
5 $2 = 2x - 14$	5 Substitution
6 $+14 \quad +14$	6 Addition Prop.
7 $16 = 2x$	7 Substitution
8 $\frac{16}{2} = \frac{2x}{2}$	8 Division Prop.
9 $8 = x$	9 Substitution

10 $m\angle 3 = 6(8) + 2 = 48 + 2 = 50$ 10 Substitution

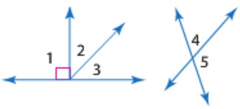
$m\angle 4 = 8(8) - 14 = 64 - 14 = 50$

Theorems Right Angle Theorems

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. $\angle$s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overrightarrow{AC} \perp \overrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. $\angle$s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. $\angle$s.	

Example 8:

Find the measure of each numbered angle, and name the theorems that justify your work.



a) $m\angle 2 = 26$

$m\angle 1 = 90$ (Thm. 2.9)

$m\angle 3 = 90 - 26 = 64$

(complements Thm)

b) $m\angle 2 = x$, $m\angle 3 = x - 16$

$m\angle 2 = 53$ $m\angle 3 = 53 - 16 = 37$

$m\angle 1 = 90$ (Thm 2.9)

$m\angle 2 + m\angle 3 = 90$

$x + x - 16 = 90$ (complements Thm)

$2x - 16 = 90$

$2x = 106$ $x = 53$

c) $m\angle 4 = 2x$, $m\angle 5 = x + 9$

supplements thm.

$m\angle 4 + m\angle 5 = 180$

$2x + x + 9 = 180$

$3x + 9 = 180$

$3x = 171$

$x = 57$

$m\angle 4 = 2(57) = 114$

$m\angle 5 = 57 + 9 = 66$

d) $m\angle 4 = 3(x - 1)$, $m\angle 5 = x + 7$

supplements thm.

$m\angle 4 + m\angle 5 = 180$

$3(x-1) + x + 7 = 180$

$3x - 3 + x + 7 = 180$

$4x + 4 = 180$

$4x = 176$

$x = 44$

$m\angle 4 = 3(44-1) = 3(43) = 129$

$m\angle 5 = 44 + 7 = 51$